

# Adjusted closing prices

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## ABSTRACT

Historical returns depend on historical closing prices and distributions. We describe how to compute adjusted closing prices from closing price/distribution data with an emphasis on spreadsheet implementation. Then the growth of a security from one date to another ( $1 + \text{total return}$ ) is just the ratio of the corresponding adjusted closing prices.

# 1 The Growth Ratio

If  $C_-$  and  $C_+$  are the closing prices of a given security on two successive business days, then the value of the security grows by  $\sigma_+ - 1$  (decimal, percent) over the second business day, where  $\sigma_+ = C_+/C_-$ . Here we assume that the second business day is not an ex-day corresponding to a declared distribution or split.

If day two is an ex-dividend day for a distribution of  $D_+$  dollars per share, then the security starts from an effective price of  $C_- - D_+$  at the end of the first day and grows to the  $C_+$  price over the course of day two. In this case the *growth ratio*  $\sigma_+$  should be defined as

$$\sigma_+ = \frac{C_+}{C_- - D_+} \quad (1d)$$

with the percentage growth again being  $\sigma_+ - 1$ . Of course equation (1d) can apply to a non-ex-day as well. Just set  $D_+$  equal to zero. After an  $\alpha:1$  split the effective starting price is  $C_-/\alpha$ , and the growth ratio is

$$\sigma_+ = \alpha \times \frac{C_+}{C_-} . \quad (1s)$$

Splits can be also thought of as distributions. An  $\alpha:1$  split corresponds to a  $D_+ = \frac{\alpha-1}{\alpha} \times C_-$  distribution.

# 2 Adjusted Closing Prices

Start with sequences of successive closing prices  $C_i$  ( $i = 0, 1, \dots, n$ ) and distributions  $D_i$  ( $i = 1, \dots, n$ ) of a given security. Define the corresponding growth ratios by

$$\sigma_i = \frac{C_i}{C_{i-1} - D_i} \quad (i = 1, \dots, n). \quad (1)$$

We will refer to any sequence of positive numbers  $x_i$  ( $i = 0, 1, \dots, n$ ) as *adjusted closing prices* for the security if

$$\frac{x_i}{x_{i-1}} = \sigma_i \quad \text{for } i = 1, \dots, n . \quad (2)$$

Then the *growth ratio* of the security for the duration under consideration is

$$\sigma = \sigma_1 \times \dots \times \sigma_n = \frac{x_n}{x_0} , \quad (3)$$

and the percentage growth or total return is  $\sigma - 1$ .

See [Investopedia](#) for a less abstract description of how to compute the adjusted closing price of a stock.

### 3 Dividend Reinvestment

If one buys  $s_0$  shares of a security at closing price  $C_0$ , then the value of the investment is  $s_0C_0$ . Suppose that dividends are reinvested automatically and that the number of shares and the closing prices over the next  $n$  business days are  $s_i$  and  $C_i$  ( $i = 1, \dots, n$ ), respectively. Then the growth ratio  $\sigma_i$  of the investment over the  $i$ -th business day should clearly be the ratio of the value of the investment at the close of day  $i$  to its value at the close of day  $i - 1$ :

$$\sigma_i = \frac{s_i C_i}{s_{i-1} C_{i-1}} . \tag{4}$$

By equating the  $\sigma_i$  of (1) with the  $\sigma_i$  of (4) we see that

$$s_i = s_{i-1} + \Delta s_i , \quad \Delta s_i = \frac{D_i}{C_{i-1} - D_i} . \tag{5}$$

Thus the new shares  $\Delta s_i$  are precisely what can be purchased with the dividend  $D_i$  at the “ex-closing-price”  $C_{i-1} - D_i$ .

### 4 Spreadsheet Considerations

Suppose successive closing prices  $C$ , distributions  $D$ , and growth ratios  $\sigma$  (as computed by (1)) are recorded as three columns of a spreadsheet. The formulas

$$x_+ = x_- \times \sigma_+ \tag{FILL+}$$

$$x_- = x_+ \div \sigma_+ \tag{FILL-}$$

then allow one to fill in adjusted closing prices from an arbitrary base adjusted closing price on any given business day. If the  $+ =$  time direction is up, enter the right-hand formula of the (FILL+) equation in the cell above the base price (now  $x_-$ ) and “Fill Up” from there, and enter the right-hand formula of the (FILL-) equation in the cell below the base price (now  $x_+$ ) and “Fill Down” from this cell. In this way all adjusted closing prices can be in filled in, up or down, from the base adjusted closing price. (If the  $+ =$  time direction is down, one should “Fill Down” with FILL+ and “Fill Up” with FILL-.)

The following spreadsheet image illustrates the procedure. The adjusted closing price of iShares Barclays 1-3Yr Treasury Bond Fund (SHY) is arbitrarily set at 100.000 on December 14, 2007. All other adjusted closing prices are filled up or down from this base price using (FILL+) or (FILL-), respectively. (The empty distribution cells are computed as zero.)

Figure 1: Filling a spreadsheet

	A	B	C	D	E
1	shy_data.csv: price-distribution data for				
2	SHY: iShares Barclays 1-3Yr Treasury Bond Fund				
3		closing		growth	adjusted
4	date	price	distrib	ratio	close
602	2007-12-31	82.19		1.0016	100.72
603	2007-12-28	82.06		1.0012	100.56
604	2007-12-27	81.96	0.2794	1.0022	100.44
605	2007-12-26	82.06		0.9990	100.22
606	2007-12-24	82.14		0.9991	100.32
607	2007-12-21	82.21		0.9984	100.40
608	2007-12-20	82.34		1.0007	100.56
609	2007-12-19	82.28		1.0026	100.49
610	2007-12-18	82.07		0.9994	100.23
611	2007-12-17	82.12		1.0029	100.29
612	2007-12-14	81.88		0.9976	100.00
613	2007-12-13	82.08		0.9985	100.24
614	2007-12-12	82.20		0.9975	100.39
615	2007-12-11	82.41		1.0037	100.65
616	2007-12-10	82.11		0.9993	100.28
617	2007-12-07	82.17		0.9989	100.35
618	2007-12-06	82.26		0.9979	100.46
619	2007-12-05	82.43		1.0004	100.67
620	2007-12-04	82.40		0.9999	100.64
621	2007-12-03	82.41	0.2760	1.0023	100.65
622	2007-11-30	82.50		1.0005	100.42

The growth of SHY in December 2007 is measured by the ratio of its 2007-12-31 adjusted closing price to its 2007-11-30 price,

$$\frac{100.72}{100.42} = 1.003.$$

Thus SHY gained 0.3% in December 2007. The ratio, 1.003, is also the product of the December growth ratios:  $1.0023 \times \dots \times 1.0016$ .

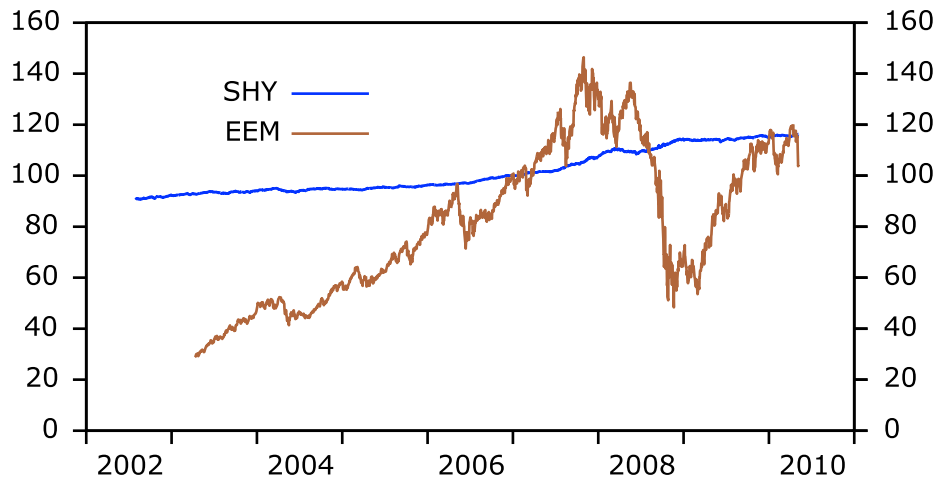
## 5 Data

Here are two comma-separated-value (spreadsheet) files, [shy\\_data.csv](#) and [eem\\_data.csv](#), with closing price, distribution, and adjusted closing price data for

SHY: iShares Barclays 1-3Yr Treasury Bond Fund  
 EEM: iShares MSCI Emerging Markets Index Fund

respectively. The graphs of these adjusted closing price data are shown in Figure 2.

Figure 2: Adjusted closing prices  
with base price = 100 on 2006-12-29



We have chosen the bond fund SHY because has lots distributions to work with, generally one per month. On the other hand the equity fund EEM illustrates how splits are handled; the shares of EEM split 3 for 1 in 2005 and again in 2008.

One can get historical closing prices for any security at [Yahoo! FINANCE](http://finance.yahoo.com) or more specifically, for our funds, at

<http://finance.yahoo.com/q/hp?s=SHY+Historical+Prices>

<http://finance.yahoo.com/q/hp?s=EEM+Historical+Prices>

Dividends and splits can be found here as well, but unfortunately some dividends may be missing. For example Yahoo! FINANCE is missing the 27-Dec-2007 SHY dividend of \$0.2794 per share shown in Figure 1.

Yahoo! FINANCE also gives adjusted closing prices, but, because of missing dividends, these prices may be inaccurate. Consider SHY for example. The Yahoo's 2007-12-31 adjusted closing price is less than its 2007-11-30 adjusted closing price. Thus Yahoo! FINANCE would have you believe that SHY lost money in December 2007, when, in fact, SHY gained 0.3% in this month—as we have just seen.

The best place to get iShares distribution data is from the horse's mouth: go to

[http://us.ishares.com/product\\_info/fund/](http://us.ishares.com/product_info/fund/)

click on the desired fund and then on the Distributions tab. In our case

[http://us.ishares.com/product\\_info/fund/distributions/SHY.htm](http://us.ishares.com/product_info/fund/distributions/SHY.htm)

[http://us.ishares.com/product\\_info/fund/distributions/EEM.htm](http://us.ishares.com/product_info/fund/distributions/EEM.htm)

will get you all distributions of SHY and EEM, respectively.

## 6 Closing Remarks

All of this material probably appears elsewhere. We just don't know where.

## References

Investopedia. How do I calculate the adjusted closing price for a stock? <<http://www.investopedia.com/ask/answers/06/adjustedclosingprice.asp>>.